RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. SIXTH SEMESTER EXAMINATION, MAY-JUNE 2013

THIRD YEAR

Date : 28/05/2013

Mathematics (Honours)

Time : 11am – 3pm

Paper : VIII

Full Marks : 70

(Use separate answer books for each Group)

Group - A

Answer **any five** questions from the following :

1. Show that three coplanar forces P_1, P_2, P_3 acting at points A_1, A_2, A_3 respectively are in astatic equilibrium if they meet in a point O situated on the circumcircle of A_1, A_2, A_3, and if $P_1 : P_2 : P_3 = A_2A_3 : A_3A_1 : A_1A_2$. [6]

2. A rough wire in the form of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is placed with its x-axis vertical and y-axis horizontal. If μ be the coefficient of friction, find the depth below the highest point of the position of limiting equilibrium of a bead which rests on the wire. [6]

3. A rod AB is movable about a joint at A, and to B is attached a string whose other end is tied to a ring. The ring slides along a smooth horizontal wire passing through A. Prove by the principle of virtual work, that the horizontal force necessary to keep the ring at rest is $\frac{W\cos\alpha\cos\beta}{2\sin(\alpha+\beta)}$, where W is the weight of the rod and α,β the inclinations of the rod the string to the horizontal. [6]

4. Discuss the stability of equilibrium of a system of rigid bodies when gravity is the only external force. [6]

5. A stiff wire in the form of a parabola rests on the horizontal ground with its plane vertical. The centre of gravity of the wire is on the axis of the parabola at a distance h from the vertex and the latus rectum is 4a. Prove that, if h>2a, there is a position of equilibrium in which the axis makes an angle $\tan^{-1}\sqrt{\frac{a}{h-2a}}$ with the horizontal and that this position of equilibrium is stable. [6]

6. When a curve is said to be a catenary of uniform strength?
The distance between the points of support in the same horizontal line of a catenary of uniform strength is 'a' and the length of the chain is 'ℓ'. Show that the parameter 'c' can be determined from the equation $\tanh\frac{\ell}{4c}=\tan\frac{a}{4c}$. [1+5]

7. Show that any system of forces acting on a rigid body can be reduced to a single force and a couple whose axis lies on the line of action of the force. [6]

8. A force P acts along the axis of x and another force 2P acts along a generator of the cylinder $x^2 + y^2 = a^2$. Show that the central axis lies on the cylinder $4(2x-z)^2 + 25y^2 = 16a^2$. [6]

Group - B

Answer **any two** questions from the following :

9. a) Assuming 5 –bit binary number with left most bit being the sign bit, perform the following subtraction using 2's complement: 00101 – 00100. [3]
   b) Find the number N such that $(11011)_2 \times (11101)_2 = (N)_8$. [3]
   c) Write an algorithm to find the H.C.F. of two distinct positive integers by Euclid's algorithm,

indicating also the case when the numbers are coprime. [4]

10. a) In a Boolean Algebra B, prove that a + (b + c) = (a + b) + c $\forall a, b, c \in B$. [3]

b) f is a function of three Boolean variables x, y, z defined by f (x, y, z) = xy + z'. Express f(x, y, z) in disjunctive normal form. [3]

c) A committee of three persons A, B, C decides proposals by a majority of votes. A has a voting weight 3, B has a voting weight 2 and C has a voting weight 1. Design a simple circuit so that light will glow when a majority of votes is cast in favour of the proposal. [4]

11. a) Write an efficient computer program in C language to sort the following set of real numbers in ascending order : 10.2, 14.6, 3.9, 8.6, 5.8, 13.5, 2.4, 7.5, 4.5 and 11.2. [5]

b) Write a C program using switch statement to determine roots of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$, where a, b, c are given. [5]

Group – C

[Answer either Unit-I or Unit-II]

Unit - I

Answer **any four** questions :

12. a) Prove that the components of a tensor of type (0, 2) can be expressed as the sum of a symmetric tensor and a skew symmetric tensor of same type. [3]

b) If a tensor A_{ijkl} is symmetric in the first two indices from the left and skew symmetric in the second and fourth indices from the left, show that $A_{ijkl} = 0$. [2]

13. Prove that there is no distinction between covariant and contravariant vectors under transformation of the form $\bar{x}^i = a^i_m x^m + b^i$, where a^i_m and b^i are constants such that $a^i_r a^i_m = s^r_m$ (i summed). [5]

14. If the metric is given by $(ds)^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$, evaluate (i) g , and (ii) g^{ij}. [5]

15. Define a unit vector in Riemannian space V_n. Show that in V_4 with line element ds defined by $ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$, the vector $(\sqrt{2}, 0, 0, \frac{\sqrt{3}}{c})$ is a unit vector. [1+4]

16. a) Prove that $\dfrac{\partial g^{ik}}{\partial x^j} = -g^{hk}\begin{Bmatrix} i \\ h\ j \end{Bmatrix} - g^{hi}\begin{Bmatrix} k \\ h\ j \end{Bmatrix}$. [4]

b) Prove that $\delta\,{}^{\;i}_{j,k} = 0$. [1]

17. Define Christoffel symbol of the second kind. Show that $\begin{Bmatrix} i \\ i\ j \end{Bmatrix} = \dfrac{\partial \log\sqrt{g}}{\partial x^j}$, where $g = |g_{ij}| \neq 0$. [1+4]

Unit – II

Answer **any four** questions :

12. Define regular value of a differentiable map $f : U \to \mathbb{R}$, where $U \subseteq \mathbb{R}^3$ is an open set. State the implicit function theorem and use it to show that if $a \in \mathbb{R}$ is a regular value of the above map f & if the set $S = f^{-1}(\{a\})$ is non empty, them S is a surface. [1+4]

13. Define unit normal vector field on a surface S. Show that if S is a connected surface & N_1, N_2 be two unit normal vector fields on S, there either $N_1 = N_2$ or $N_1 = -N_2$. [1+4]

14. Show that the sphere is an orientable surface. Compute the Gauss and Mean curvature of sphere. [2+3]

15. Define curvature $K_\alpha(s)$ of a curve $\alpha : I \to \mathbb{R}^2$ (parametrized by arc length) at a point $s \in I$. Let $\alpha : \mathbb{R} \to \mathbb{R}^2$ be a curve defined by $\alpha(s) = c + r\left(\cos\frac{s}{r}, \sin\frac{s}{r}\right)$ for some $c \in \mathbb{R}^2$ & r>0. Compute $K_\alpha(s)\ \forall s \in \mathbb{R}$. [2+3]

16. Let $I \subseteq \mathbb{R}$ be an open interval and $K_0, \widehat{C}_0 : I \to \mathbb{R}$ be two differentiable functions with $K_0(s) > 0\, \forall s \in I$ Then $\exists$ a curve $\alpha : I \to \mathbb{R}^3$ parametrized by arc length such that $K_\alpha(s) = K_0(s)$ & $\tau_\alpha(s) = \tau_0(s)\ \forall s \in I$ where $K_\alpha\ \&\ \tau_\alpha$ are the curvature and torsion function of α. Furthermore α is unique upto a direct rigid motion of Euclidean space $\mathbb{R}^3$. [3+2]

17. Let S be a surface, $f : S \to \mathbb{R}$ be a differentiable function and $P \in S$ be a regular point of f. Show that $\exists$ an open neighbourbood V of P in S, a real no. $\varepsilon > 0$ and an injijective regular curve $\alpha : (-\varepsilon, \varepsilon) \to \mathbb{R}^3$ which is homeomorphic onto its image such that $\alpha(0) = P$ and $f^{-1}(\{a\}) \cap V = \alpha(-\varepsilon, \varepsilon)$, where f(P) = a. [5]